

Quiz 9 – 11/22/2023

Instructions. You have the entire class to complete this quiz. You may use your own course materials, as well as any materials directly linked from the course website. You may use your plebe-issue calculator. You may collaborate with other students on this quiz.

Show all your work. To receive full credit, your solutions must be completely correct, sufficiently justified, and easy to follow.

Problem	Weight	Score
1	1	
2	0.5	
3	0.5	
Total		/ 20

Problem 1. Customers call the reservation desk at Fluttering Duck Airlines at a rate of 4 customers per hour. There is 1 agent working at the reservation desk at any given time, and each phone call takes an average of 20 minutes. The phone system can only handle 3 customers at a time (1 with an agent, 2 waiting) – any phone calls arriving when there are 3 customers are simply lost.

This setting can be modeled as a birth-death process with the following arrival and service rates:

$$\lambda_i = \begin{cases} 4 & \text{if } i = 0, 1, 2 \\ 0 & \text{if } i = 3, 4, \dots \end{cases} \quad \mu_i = 3 \quad \text{for } i = 1, 2, \dots$$

- a. Over the long run, what is the probability that there are n customers in the system? ($n = 0, 1, 2, 3$)

b. Suppose you find that the steady-state probabilities are:

$$\pi_0 = 0.15 \quad \pi_1 = 0.21 \quad \pi_2 = 0.27 \quad \pi_3 = 0.37 \quad \pi_n = 0 \quad \text{for } n = 4, 5, \dots$$

(This may or may not match what you found in part a.) Over the long run, what is the expected number of customers in the system?

c. Suppose you find that the long-run expected number of customers in the system is 1.86. (This may or may not match what you found in part b.) Over the long run, what is the expected waiting time? Use the steady state probabilities given above in part b.